A New Transmission Line of Round Conductor and Parallel Plane with Symmetrically Placed Slit

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Abstract —**A new transmission line consisting of a cylindrical conductor symmetrically placed opposite a slit in an infinite flat plate is formed by inversion from the line of parallel plates of equal width. The characteristic admittance of the proposed line and one possible application of which is given.**

To solve the problem of the characteristic admittance (or the capacitance) of a new transmission line consisting of a circular cylinder of radius R_0 symmetrically placed opposite a slit of width $2s$ in an infinite flat plate, as shown in Fig. 1, we extend the method of inversion applied to the parallel-plate capacitor with a symmetrically placed equal plates [1]. We now take the following bilinear transformation

$$z = \frac{Az_1 + B}{z_1 + C} = A + \frac{H}{z_1 + C} \tag{1}$$

where

$$C = C_1 + jC_2 \quad H = H_1 + jH_2$$
$$A = A_1 + jA_2 \quad B = B_1 + jB_2$$

are constants, to transform the upper z-plane onto the upper z_1-plane of Fig. 2 so that the line segment

$$y_1 = x_1 + jh, \qquad |x_1| \leqslant a$$

is transformed into the real axis $y = 0$ with a slit of width $2s$ and the real axis $y_1 = 0$ is transformed into a circle of radius R_0.

Firstly, we eliminate $x_1 + C_1$ from (1) to obtain the equation of a circle in the z-plane of Fig. 1

$$\left[x - \left(A_1 + \frac{H_2}{2}\frac{1}{y_1 + C_2}\right)\right]^2 + \left[y - \left(A_2 - \frac{H_1}{2}\frac{1}{y_1 + C_2}\right)\right]^2 = \frac{|H|^2}{4(y_1 + C_2)^2} = R_0^2$$

and by comparing the points $z_1 = \pm jh$ and $z = a_1$, a_2 and the line $y_1 = 0$ with a circle in the z-plane in Fig. 1, we obtain then

$$A = \frac{a_1 + a_2}{2} + jo \quad B = \frac{a_2 - a_1}{2}a - j\frac{a_2 + a_1}{2}h$$
$$C = 0 - jh \qquad H = \frac{a_2 - a_1}{2}a + jo$$

and the circle in the z-lane goes into

$$(x - A_1)^2 + \left(y + \frac{H_1}{2C_2}\right)^2 = \frac{H_1^2}{4C_2^2} = R_0^2. \tag{2}$$

Secondly, we take for simplicity $a_1 + a_2 = 0$. Then $A_1 = A_2 = 0$, $H_1 = a_2 a$, and (2) becomes

$$x^2 + \left(y - \frac{a}{h}\frac{a_2}{2}\right)^2 = \left(\frac{a}{h}\frac{a_2}{2}\right)^2 = R_0^2 \tag{3}$$

so that when $x = 0$

$$y = 0 \quad \text{and} \quad y = \frac{2H_1}{2(-C_2)} = \frac{a}{h}a_2.$$

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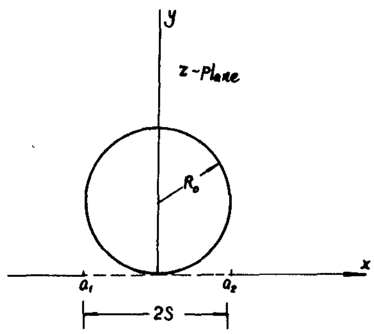

Fig. 1.

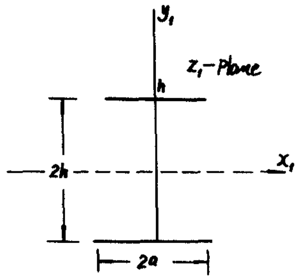

Fig. 2.

Therefore, we obtain the slit width

$$2s = 2a_2 \tag{4}$$

and the radius of the cylinder R_0 from (3)

$$R_0 = \frac{a}{h}\frac{a_2}{2} = \frac{a}{h}\frac{s}{2}$$

so

$$\frac{s}{R_0} = 2\frac{h}{a}. \tag{5}$$

The characteristic admittance of the line of Fig. 1 is given by

$$Y_0 = \sqrt{\frac{\epsilon}{\mu}}\;\frac{C}{\epsilon} = \frac{C}{\sqrt{\mu\epsilon}}$$

where μ and ϵ are the absolute permeability and permittivity of the medium surrounding the transmission-line conductor in Fig. 1, and C is the capacitance per unit length of the line and is just the capacitance between one plate of width $2a$ and the zero potential plane $y_1 = 0$ of Fig. 2, or

$$\frac{C}{\epsilon} = 2\frac{K(k')}{K(k)} \tag{6}$$

where $k' = \sqrt{1 - k^2}$ and $K(k)$ and $K(k')$ are complete elliptic integrals of the first kind of modulus k and its compliment k', respectively. Equation (6) can be calculated more readily from relations

$$\frac{a}{h} = \frac{2K'}{\pi}\left\{\sqrt{1 - \frac{E'}{K'}}\sqrt{1 - k^2}\frac{K'}{E'} - z(v, k')\right\} \tag{7}$$

given in [3] (the expression of b in [2] seems to be wrong), where $Kz(v, k)$ is tabulated in [2] and v is related to k by

$$\frac{E(k')}{K(k')} = k\,\mathrm{sn}(K + jv) = \frac{k}{\mathrm{dn}(v, k')} \tag{8}$$

where $E(k)$ is the complete elliptic integral of the second kind.

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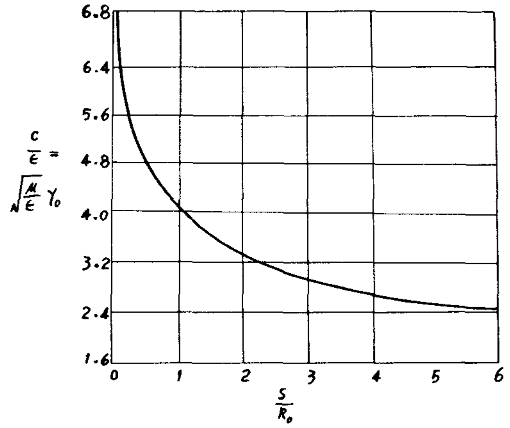

Fig. 3.

We finally have (8) and the following equation:

$$\frac{s}{R_0} = \frac{\pi}{2K'\left\{\sqrt{1-\frac{E'}{K'}}\,\sqrt{1+k^2}\,\frac{K'}{E'} - z(v,k')\right\}} \tag{9}$$

to solve for k and v simultaneously to substitute the former into (6) to find C. Conversely, given k, we can calculate C from (6) and the ratio of slit width $2s$ to the diameter $2R_0$ by (8) and (9) of Fig. 1. The curve in Fig. 3 is constructed from [1].

This new transmission line of Fig. 1 is of reduced height, which is equal to the diameter of its upper circular conductor, and intuitively this new line will have lower loss than the conventional two-wire line because its large plane conductor will offer low ohmic, as well as radiation, loss. In addition, it can be used to detect and to measure the width of the slit of a flat conducting plate, because when $2R_0$ is known, the width $2s$ of the slit can be calculated from the measured value of C, from the curve of Fig. 3 or from (6).

References

[1] N. N. Langton, "The parallel-plate capacitor with symmetrically placed unequal plates," 289–306, *J. Electrostatics*, vol. 12, no. 4, pp. 289–306, June 1981.

[2] P. F. Byrd and M. D. Friedman, *Handbook of Elliptic Integrals for Engineers and Scientists*. Berlin Heidelberg, New York: Springer-Verlag 1971, 119.03.

[3] K. J. Binns and P. J. Lawrenson, *Analysis and Computation of Electric and Magnetic Field Problems*. Oxford: Pergamon Press, 1963, p. 315.

Letters

Correction to "Optical Fiber Delay-Line Signal Processing"

Due to a clerical error, the above paper[1] by K. P. Jackson, S. A. Newton, B. Maslehi, M. Tur, C. C. Cutler, J. W. Goodman and H. J. Shaw appeared in the March 1985 issue (pp. 193–210) without being identified as an *Invited Paper*.

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[1] K. P. Jackson *et al.*, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 193–210, Mar. 1985.

Comments on "Scattering at a Junction of Two Waveguides with Different Surface Impedances"

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In the above paper,[1] a criterion has been given in order to establish if the scattering problem of the junction between two waveguides with different surface impedances can be solved in closed form. In this comment, a different approach, based on a spectral formulation, shows that the possibility to obtain analytical expressions of the scattering coefficients depends on the form of the relevant Wiener–Hopf equation.

The above paper[1] presents some results on the problem of the scattering at the junction of two waveguides having different surface impedances. From a theoretical point of view, the most important concerns the possibility to obtain analytical expressions of the scattering coefficients when certain conditions on the geometries of the waveguides are satisfied. The procedure used in [1] has some limitations; more general results can be obtained by following a different approach based on the Wiener–Hopf formulation of the problem. Let us consider [1, fig. 1] and indicate with a and a' the waveguides at the left and the right side, respectively, of the junction. A spectral formulation of the problem leads to a Wiener–Hopf equation having the form

$$G(\alpha)F_+(\alpha) = F_-(\alpha) + F_0(\alpha) \tag{1}$$

where $G(\alpha)$ and $F_0(\alpha)$ are known, and the unknowns $F_+(\alpha)$ and $F_-(\alpha)$ are the Fourier transforms of suitable components of the electromagnetic fields in the guide a' and a, respectively. In all

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[1] C. Dragone, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1319–1328, Oct. 1984.

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